TN Department of Education

# Eighth Grade Mathematics

# Instructional Focus Documents

## Introduction:

The purpose of this document is to provide teachers a resource which contains:

- The Tennessee grade-level mathematics standards
- Evidence of Learning Statements for each standard
- Instructional Focus Statements for each standard

## Evidence of Learning Statements:

The evidence of learning statements are guidance to help teachers connect the Tennessee Mathematics Standards with evidence of learning that can be collected through classroom assessments to provide an indication of how students are tracking towards grade-level conceptual understanding of the Tennessee Mathematics Standards. These statements are divided into four levels. These four levels are designed to help connect classroom assessments with the performance levels of our state assessment. The four levels of the state assessment are as follows:

- Level 1: Performance at this level demonstrates that the student has a minimal understanding and has a nominal ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 2: Performance at this level demonstrates that the student is approaching understanding and has a partial ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Level 3: Performance at this level demonstrates that the student has a comprehensive understanding and thorough ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.
- Levels 4: Performance at these levels demonstrates that the student has an extensive understanding and expert ability to apply the grade-/course-level knowledge and skills defined by the Tennessee academic standards.

The evidence of learning statements are categorized in the same way to provide examples of what a student who has a particular level of conceptual understanding of the Tennessee Mathematics Standards will most likely be able to do in a classroom setting.

## Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance around Tier I, on-grade level instruction. Thus, the instructional focus statements are written for levels 3 and 4.

# The Number System (NS)

**Standard 8.NS.A.1 (Supporting Content)**

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually or terminates, and convert a decimal expansion which repeats eventually or terminates into a rational number.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
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| Convert fractions to decimals.<br><br>Differentiate between non-repeating, non-terminating decimals and repeating decimals.<br><br>Differentiate between terminating and repeating decimals.<br><br>Convert terminating decimals to fractions. | Identify a given number as rational or irrational.<br><br>Convert repeating decimals to fractions. | Explain how irrational numbers differ from rational numbers.<br><br>Determine when the decimal expansion of a fraction will terminate or repeat.<br><br>Show that the decimal expansion of rational numbers eventually repeats or terminates. | Explain the relationship between numbers within the real number system using precise mathematical language. |

## Instructional Focus Statements

**Level 3:**

In grade 8, students perform operations with rational numbers to expand their understanding of the Real Number System by recognizing irrational numbers and their relationship to rational numbers. Students learn that numbers in the Real Number System are either rational or irrational and develop an understanding that when an irrational number is converted to a decimal, it is non-repeating and non-terminating. Likewise, when a rational number is converted to a decimal, it will repeat or terminate. To foster these understandings, students should access prior knowledge of the long division algorithm from earlier grades to convert fractions to decimals. Instruction should also include opportunities recognize numbers such as square roots of non-perfect squares and pi into approximated decimals. Discussion should focus students to determine when the conversion results in a repeating or non-terminating decimal and students should be expected to use the proper notation for expressing these values. Additional practice converting fractions to decimals and discussion around the factors of their denominators will help as students eventually learn to predict when the decimal expansion of a fraction will be terminating. Just the same, students should engage in discussions around repeating patterns that occur when fractions have denominators of 9, 99 or 11. Discussion should be facilitated so that students eventually discover that the decimal expansion of irrational numbers can only be approximated.

Instruction should build on what students already know about the hierarchy of the Real Number System. As students are introduced to irrational numbers and extend their understanding of rational numbers to include non-perfect squares and non-terminating decimals, a visual representation or diagram will aid in their understanding of how rational and irrational numbers make up the real number system and how each subset of rational numbers relate to each other.

Students should also engage in discourse that leads to the understanding of why irrational numbers cannot be written as rational numbers. A common misconception is that all numbers in fractional form are rational. Therefore, students should be exposed to irrational numbers in fractional form such as $\frac{\pi}{2}$ and engage in discourse around fractional form and rational form.

**<u>Level 4:</u>**

Students at this level go beyond determining when a number is rational or irrational. They understand that real numbers are either rational or irrational and can explain why and how they make up the Real Number System. Students should be challenged to justify their explanations with illustrations and graphing organizers that include examples of rational and irrational numbers. In addition to discussing the relationship between rational and irrational numbers, students should be expected to explain the relationship between various types of rational numbers as well.

**Standard 8.NS.A.2 (Supporting Content)**

Use rational approximations of irrational numbers to compare the size of irrational numbers locating them approximately on a number line diagram. Estimate the value of irrational expressions such as $\pi^2$ . For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Plot positive rational numbers on the number line.<br><br>Order positive rational numbers using the number line.<br><br>Compare positive rational using the number line.<br><br>Make comparative statements about the size of positive rational numbers.<br><br>Evaluate numerical expressions containing positive rational numbers.<br><br>Write the decimal equivalent of rational numbers. | Plot rational numbers on the number line.<br><br>Compare rational numbers using the number line.<br><br>Order rational numbers using the number line.<br><br>Make comparative statements about the size of rational numbers.<br><br>Evaluate numerical expressions containing rational numbers.<br><br>Write the decimal approximation of irrational numbers. | Estimate the value of irrational numbers using rational approximations.<br><br>Compare real numbers using the number line.<br><br>Plot real numbers on the number line using their estimated values.<br><br>Order real numbers using the number line.<br><br>Make comparative statements about the size of irrational numbers.<br><br>Estimate the value of irrational expressions. | Explain how to arrive at a more precise approximation when given an approximation of an irrational number. |

## Instructional Focus Statements

**Level 3:**

In grade 6 students located rational numbers on the number line and in grade 7, students converted rational numbers to decimals using long division. In grade 8, students build on this learning to estimate values of irrational numbers and compare real numbers by positioning them on a number line. Using prior knowledge of the size of rational numbers, students should have opportunities to demonstrate finding rational approximations for irrational numbers such as $\sqrt{10}$. When estimating the value of an irrational number, discussion should include reasoning with the use of slightly larger or smaller rational numbers. In this case, they would explain that $\sqrt{10}$ is in between the two perfect squares $\sqrt{9}$ and $\sqrt{16}$, implying that $\sqrt{10}$ is between 3 and 4 and closer to, yet a little bit more than 3. Additional discussion should lead to the discovery of a more precise approximation as students estimate within which tenth the number lies, and then within which hundredth it lies.

Students should have multiple opportunities to plot real numbers on the number line and justify placement of the numbers using appropriate mathematical language. Students should also explain when a number's location should be approximated and justify comparisons of irrational numbers in relation to rational numbers. Placing irrational numbers on the number line will support students' understanding of standard 8.NS.A.1, where they learned that in addition to the rational numbers they previously worked with, irrational numbers are also part of the Real Number System. Students should be encouraged to refer to the number line as the real number line to emphasize the idea that there are an infinite number of real numbers represented by the line.

While students evaluated rational expressions in previous grades, they should now be exposed to expressions that contain irrational numbers such as $2\pi$. Discussion should be facilitated in a manner that leads students to understand that the values of these expressions can only be estimated. Discussions should also include opportunities to construct viable arguments by justifying the estimated values of the irrational expressions. For example, knowing $3^2$ is 9, students could argue that $\pi^2$ is slightly greater than 9. This new understanding of irrational numbers will be essential as students progress to grade 8 geometry and use $\pi$ to find the area or circumference of a circle, or volume of a sphere. Students will also apply this understanding in grade 8 to solve problems involving the Pythagorean Theorem and non-perfect squares. Beyond grade 8, a thorough understanding of the real number system will support students' learning in high school as they begin to study complex numbers and encounter solutions that are not part of the Real Number System.

**Level 4:**

Students at this level can not only estimate values of irrational expressions, but can explain verbally and in writing how to improve a given estimation of the irrational expression. Given an irrational number, students can use a series of rational numbers to arrive at more and more precise estimations for the irrational number and can communicate this process using precise mathematical language. Students at this level should have opportunities to analyze an estimation of irrational numbers and discuss the process for extending the decimal to the hundredth, thousandth or further place to get a more precise estimation of the number.

## Expressions and Equations (EE)

**Standard 8.EE.A.1 (Major Work of the Grade)**

Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \, x \, 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

### Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
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| State that negative numbers raised to an even power can yield different products when parentheses are used. For example, $(-5)^2 = 25$ and $-5^2 = -25$.<br><br>State that any number raised to the zeroth power has a value of 1.<br><br>Write repeated multiplication of rational numbers using exponential notation. | Use the product rule with positive integer exponents to write equivalent numerical expressions using exponential notation.<br><br>Use the quotient rule with positive integer exponents to write equivalent numerical expressions using exponential notation.<br><br>Re-write a numerical expression, involving multiplication or division with positive integer exponents, as a simplified integer. | Use properties of integer exponents to generate equivalent numerical expressions (e.g., product rule, quotient rule, power rule, power of a product rule, zero exponent rule, and negative exponent rule).<br><br>Rewrite numerical expressions with fractional bases raised to a power. | Explain why finding the power of a product results in each factor being raised to that power.<br><br>Explain why any number to the zeroth power is 1 using known properties of integers and precise mathematical language.<br><br>Explain how the properties of exponents apply to negative exponents. |

### Instructional Focus Statements

**Level 3:**

In grades 6 and 7 students wrote and evaluated numerical expressions with whole number exponents. They became familiar with exponential notation and learned that a number raised to a power (exponent) was a symbolic way to represent the repeated multiplication of that number. In grade 8, students continue to evaluate expressions with exponents, but now the exponents can be both positive and negative integers.

As students extend this thinking to develop an understanding of the properties of exponents, they should have opportunities to explore by expanding numerical expressions and observing patterns. Instruction should not focus on merely memorizing properties of exponents (or rules). Students should discover the properties through repeated reasoning. For example, expressions involving multiplication or division with like bases, $3^2 \cdot 3^5 = 3^7$. Multiple opportunities to practice expanding the factors $((3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3))$ will foster students' understanding of why we can add the exponents when multiplying with like bases and why the two expressions are equivalent. ($3^2 \cdot 3^5 = 3^{2+5} = 3^7$). Students learn that a power of a power can be found by multiplying the exponents. For example, $(-6^2)^4 = (-6 \cdot -6) \cdot (-6 \cdot -6) \cdot (-6 \cdot -6) \cdot (-6 \cdot -6) = -6^8$. It is a common misconception for students to add these exponents, multiple opportunities to explore will aid in minimizing this misconception. This same practice is essential for understanding how each of the properties work. Though each property is unique, students should be challenged to apply multiple properties and re-write more complex expressions, for example, $\frac{(4^5)^{10} \cdot 4^{200}}{(4^2)^{20}}$. Furthermore, students should engage in discourse around the idea that there can be multiple expressions equivalent to the original one. Students will benefit from a solid understanding of these properties when they later explore scientific notation and make sense of very large and small numbers in standards 8.EE.A.3 and 8.EE.A.4.

**Level 4:**

At this level, students should be challenged to go beyond knowing and applying the properties of integer exponents. They should engage in discourse that prompts them to prove certain knowns related to integer exponents. For example, when finding the power of a product, students should be able to explain that each factor is raised to that power. Discussion should include their use of the associative and commutative properties to justify this using student generated examples.

Students know that a number raised to the zeroth power has a value of 1. Students should be probed to prove it using what they know about dividing exponential expressions with like bases. For example, they know that $\frac{5^2}{5^2} = 5^0 = 1$ can be written as $5^{2-2}$ or $5^0 = 1$. Although students' initial understanding of exponents includes positive integer exponents, additional practice should focus on explaining why these properties can also be applied in situations with negative exponents.

**Standard 8.EE.A.2 (Major Work of the Grade)**

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

## Evidence of Learning Statements

| **Students with a level 1 understanding of this standard will most likely be able to:** | **Students with a level 2 understanding of this standard will most likely be able to:** | **Students with a level 3 understanding of this standard will most likely be able to:** | **Students with a level 4 understanding of this standard will most likely be able to:** |
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| Recognize that a positive rational perfect square number is the product of two identical factors that can be written using a power of 2.<br><br>Recognize that a positive rational perfect cube number is the product of three identical factors that can be written using a power of 3.<br><br>Determine if a number is rational or irrational. | Identify the relationship between raising a number to second power (squaring a number) and taking the square root of a number as inverse operations.<br><br>Identify the relationship between raising a number to the third power (cubing a number) and taking the cube root of a number as inverse operations.<br><br>Know that the solution to the equation $x^2 = p$ can be a positive or negative value (x = $\pm\sqrt{p}$). | Identify the square root of a non-perfect square as irrational.<br><br>Identify the cube root of a non-perfect cube as irrational.<br><br>Evaluate square roots of small perfect square numbers.<br><br>Evaluate cube roots of small, perfect cube numbers.<br><br>Solve equations that require finding the square root of a number of the form, $x^2 = p$, where $p$ is a positive rational small perfect square number.<br><br>Solve equations that require finding the cube root of a number of the form, $x^3 = p$, where $p$ is a positive rational small perfect cube number. | Solve real-world problems that involve evaluating square roots and cube roots.<br><br>Solve real-world problems that involve equations of the form $x^2 = p$ or $x^3 = p$, where $p$ is a positive rational number.<br><br>Create contextual problems that represent an equation of the form of $x^2 = p$ or $x^3 = p$, where $p$ is a positive rational number.<br><br>Explain why the square root of a non-perfect square or cube root of a non-perfect cube is irrational using precise mathematical language. |

## Instructional Focus Statements

**Level 3:**

Students used whole number exponents to denote powers of 10 using place value in grade 5. They extended their understanding of whole-number exponents to evaluate numerical expressions in grades 6 and 7. In grade 8, students should be able to identify the relationship between raising a number to the second power (squaring a number) and taking the square root of a number as inverse operations and likewise, for raising a number to the third power and cube roots. It is essential for students to understand this relationship when they solve equations of the form $x^2 = p$ or $x^3 = p$, where $p$ is a positive rational number. In solving these equations, students should be challenged to extend their understanding of the properties of equality and realize that taking the square root of one side of an equation requires taking the square root of the other side. As students solidify this understanding, they should be given opportunities to practice with positive rational numbers to discover that the same holds true when taking the cube root of one side of an equation.

As students develop a conceptual understanding of operating with square roots and cube roots, concrete visuals (e.g., square tiles and cubes) should be used to foster this understanding. Students should understand the relationship between perfect square numbers and the length of the side of a square and the same for perfect cube numbers and the length of the side of a cube. Discussion should focus on the relationship between repeated factors and expressing them as a base and an exponent. For example, the square root of 4 is 2 because the product of $2 * 2 = 4$, which can be expressed as $2^2 = 4$. These factors can also be connected to the side lengths of a square.

As students learn to evaluate square roots, they should understand that the solution to the equation $x^2 = p$ can be a positive or negative value ($x = \pm\sqrt{p}$). Discussion should be focused on the properties of multiplying negative numbers to help students determine that the evaluation of a cube root will yield one solution. In 8.NS.A.1, students learn that numbers that are not rational are called irrational numbers. Opportunities to evaluate square roots of perfect square numbers and square roots of non-perfect square numbers will support students' understanding of the relationship between rational numbers and irrational numbers. Additional practice with cube roots of perfect cube numbers and non-perfect cube numbers should be employed to solidify this understanding. Evaluating the square root of a negative number is not expected at this grade level as students will explore the concept of imaginary numbers in high school.

**Level 4:**

Once students have a strong understanding of square roots and cube roots, they should be expected to explain the process for solving equations with radicals using precise mathematical language. Instruction should challenge students to explain why the cube root of a non-perfect cube is irrational using precise mathematical vocabulary and supporting their arguments with known facts about rational numbers and/or models. Challenging students to connect what happens when a negative number is raised to a power to the effects on square and cube roots should help to push their understanding of possible positive and negative roots in either situation.

Students should be challenged to apply their understanding of operating with roots to solve real-world problems involving square roots or cube roots and provide arguments to support the practicality of their solutions. Additionally, they should be provided opportunities to create real-world problems that can be solved by writing an equation in the form of $x^2 = p$ or $x^3 = p$. For example, “A square garden has an area of 225 sq. ft. What is the length of each side?”

**Standard 8.EE.A.3 (Major Work of the Grade)**
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as* $3 \times 10^8$ *and the population of the world as* $7 \times 10^9$*, and determine that the world population is more than 20 times larger.*

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Choose a number expressed in scientific notation to represent a small or large number. | Estimate very small or very large quantities using numbers that are represented in scientific notation. | Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities.<br><br>Indicate how many times larger one number represented in scientific notation is than a second number also expressed in scientific notation. | Solve real-world and mathematical problems by comparing estimated quantities represented in scientific notation and explain the benefits for using scientific notation using precise mathematical vocabulary. |

## Instructional Focus Statements

**Level 3:**

As students develop an understanding of the properties of exponents, they should extend their understanding to be able to write very large and very small numbers in scientific notation using positive and negative exponents. Students should develop a conceptual understanding of how and why a number is written in scientific notation. Additionally, students should understand the connection between a number written in scientific notation and the properties of exponents rather than memorizing a set of procedural rules.

**Level 4:**

Students should extend their understanding of the properties of exponents by solving real-world and mathematical problems involving numbers written in scientific notation. Students should also be able to compare numbers involving scientific notation and explain the comparison between the numbers by determining how many times larger or smaller one is than the other. Additionally, students should understand and explain the benefits of using scientific notation. This standard lays the foundational coursework for the sequential standard for which students will perform operations with numbers expressed in scientific notation.

**Standard 8.EE.A.4 (Major Work of the Grade)**

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
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| Choose an expression in scientific notation that represents a decimal number and vice versa. | Perform operations with numbers expressed exclusively in scientific notation.<br><br>Express a number written in decimal form in scientific notation and vice versa. | Choose units of appropriate size expressed in scientific notation to represent measurements of very large or very small quantities.<br><br>Perform operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. | Generate units of appropriate size expressed in scientific notation to represent measurements of very large or very small quantities.<br><br>Solve real-world and mathematical problems that involve performing operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used.<br><br>Interpret scientific notation that has been generated by technology. |

## Instructional Focus Statements

**Level 3:**

Students should extend previous knowledge of writing very large and very small numbers in scientific notation to solving problems that involve performing operations with numbers in scientific notation. Students should realize that quantities in real-world problems can be expressed in scientific notation or decimal form and be used interchangeably within real -world problems. Students should also focus on the size of the measurement to choose which units are appropriate for the contextual situation.

**Level 4:**

Students should extend their understanding of performing operations with numbers in scientific notation to solving contextual problems that involve both numbers in scientific notation form and decimal form. Additionally, students should be able to perform calculations using technology that produce a solution in scientific notation and be able to interpret the solution and explain their reasoning using precise mathematical vocabulary.

**Standard 8.EE.B.5 (Major Work of the Grade)**

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
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| Identify the unit rate as the slope in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. | Choose a graph that represents a given proportional relationship. | Graph a given proportional relationship Identify the slope from a provided graph of a proportional relationship and connect it to the unit rate.<br><br>Compare two different proportional relationships represented in different ways. | Compare two proportional relationships generated from different contexts in terms of the contexts they represent.<br><br>Identify the context of the unit rate from the graph of a proportional relationship generated from a contextual problem. |

## Instructional Focus Statements

**Level 3:**

Students build on prior conceptual understanding of unit rates in grade 6 and proportional relationships in grade 7 to compare graphs, tables, and equations of proportional relationships. Students should develop a solid foundation in graphing proportional relationships and determining and interpreting unit rate as the slope of a graph. As students work with different representations when comparing the unit rates as the slope of the proportional relationships, it may be an overwhelming amount of information to analyze. Scaffolds, purposeful questioning, and pressing students to justify each slope in the proportional relationships can be used to ensure that students have a strong conceptual understanding of each relationship being compared. When making comparisons, students should be flexible in using different strategies and not memorize a set of steps as a comparison process.

**Level 4:**

Students should employ their knowledge of unit rates and proportional relationships to compare two different proportional relationships in complex situations. Students should be able to write and verbalize their explanation of unit rate as the slope of a graph in a complex problem. Additionally, students should display procedural fluency by understanding that when provided information presented in a table and information presented in equation form, the two may be easier compared by graphing both relationships. As students solidify this procedural fluency, students should be able to justify their reasoning with written and verbal explanation.

**Standard 8.EE.B.6 (Major Work of the Grade)**
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; know and derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

**Evidence of Learning Statements**

| **Students with a level 1 understanding of this standard will most likely be able to:** | **Students with a level 2 understanding of this standard will most likely be able to:** | **Students with a level 3 understanding of this standard will most likely be able to:** | **Students with a level 4 understanding of this standard will most likely be able to:** |
|---|---|---|---|
| Identify the y-intercept of a given graph. | Choose an equation in the form y = mx + b or y = mx to represent a line graphed on a coordinate plane.<br><br>Find the slope of a line, given two points on the line. | Give an equation in the form y = mx + b or y = mx to represent a line graphed on a coordinate plane.<br><br>Choose a representation demonstrating that the slope is the same between any two points on a line using similar triangles. | Explain that the slope is the same between any two points on a line using similar triangles.<br><br>Derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b. |

## Instructional Focus Statements

**Level 3:**

Students should employ prior knowledge of proportions and slope to develop a conceptual understanding of why the slope of a line remains constant. Students develop this understanding by working with similar triangles that are formed by the vertical and horizontal lines from a point on a non-vertical line. Students should be exposed to multiple representations of this concept as they develop an understanding that similar “slope” triangles have equivalent side length ratios. Additionally, they should develop an understanding that the slope is the same between any two points on a line using similar triangles. As students work with this concept, they should gain an understanding of the equation of the line, y = mx +b, where m is the slope and b is the y-intercept. Students should be able to identify the slope of a line from graphs, tables, and equations and make connections between the representations.

**Level 4:**

Students should move beyond finding the slope of a line. They should use multiple representations to demonstrate and explain why any two points on a non-vertical line generate the same slope by using similar slope triangles. Additionally, students should be able to graph the equation of a line, identify the slope, and create a visual representation that shows that the slope is the same between any two points on a line using similar triangles formed by the vertical and horizontal lines from a point on a non-vertical line. As students solidify their understanding of this concept, they should be able to derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b by making use of structure from tables, equations, and graphs.

**Standard 8.EE.C.7 (Major Work of the Grade)**

Solve linear equations in one variable.

**8.EE.C.7a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

**8.EE.C.7b** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Solve linear equations in the form x + q =r or px = r. | Determine if a given linear equation in one variable has no solution, one solution, or infinitely many solutions. Solve linear equations in the form px + q = r or p(x + q) = r. | Give examples of linear equations in one variable having one solution, infinitely many solutions, or no solution.<br><br>Solve linear equations with rational coefficients whose solutions require expanding expressions using the distributive property and collecting like terms. | Explain why a linear equation has no or infinitely many solutions.<br><br>Solve linear equations with rational coefficients whose solutions require expanding expressions using the distributive property and collecting like terms and explain the operations used in each approach. |

## Instructional Focus Statements

**Level 3:**

Students should use prior knowledge of equality properties and equivalence for solving equations to solve more complex linear equations in one variable with coefficients that include integers, fractions, and decimals can be solved by expanding expressions with the distributive property and/or combining like terms and equations with variables on both sides. Students should also determine if a linear equation results in one, zero, or infinitely many solutions. It is imperative that students develop conceptual understanding of the various solution types and how they are related to the linear equation through a discovery process as opposed to a rote set of memorized rules.

Students should begin to develop a conceptual understanding of the implications when the solution for a linear equation results in one, zero, or infinitely many solutions. Students should be able to give examples of the different solution types to linear equations.

Additionally, these standards lay the foundation for all future coursework with linear equations and set a precedent for understanding with the other function types students will experience in late courses. Thus, it is imperative for students to gain an in-depth conceptual understanding of the inner workings of linear equations.

**Level 4:**

Students should extend their understanding of solving linear equations utilizing a variety of multiple properties of operations by explaining their solution approach using precise mathematical vocabulary in verbal and written forms.

Students should solidify their conceptual understanding of what it means when the solution for a linear equation results in one, zero, or infinitely many solutions. Students should understand that when $x = a$ , there is only one solution and that substituting the value of a into the equation will result in a true equation. Students should also be able to understand that when $a = a$ , there are infinitely many solutions and substituting any number into the equation will result in a true equation. In the same manner, when $a = b$ (where a and b are different numbers), there are no solutions, and any number substituted into the equation will result in a false equation.

The culmination of these standards will be the building block for future work solving pairs of simultaneous linear equations.

**Standard 8.EE.C.8 (Major Work of the Grade)**
Analyze and solve systems of two linear equations.
**8.EE.C.8a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
**8.EE.C.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
**8.EE.C.8c** Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Graph linear equations in two variables on the coordinate plane.<br><br>Rewrite linear equations from standard form to slope intercept form using the properties of equality.<br><br>Identify points on a line as solutions to their corresponding equation.<br><br>Graph a line on the coordinate plane when given a pair of coordinates.<br><br>Solve a linear equation in standard form.<br><br>Solve a linear equation in slope-intercept form. | Determine if a linear system of equations has one solution, infinitely many solutions, or no solution when given a graph.<br><br>Determine the slope when given a linear equation in slope-intercept or standard form.<br><br>Estimate the solution when given a graph to a system of equations.<br><br>Determine if a linear system of equations has one solution, infinitely many solutions or no solution when given a table.<br><br>Write linear equations in two variables from context.<br><br>Generate a linear equation when | Analyze a system of linear equations to determine if there is one solution, no solution or infinitely many solutions.<br><br>Write pairs of simultaneous equations to represent a real-world problem.<br><br>Solve a system of linear equations graphically.<br><br>Solve a system of linear equations algebraically.<br><br>Interpret the solution for systems of linear equations in terms of a given context.<br><br>Determine the solution to the | Explain characteristics of systems that have one solution, no solution or infinitely many solutions using precise mathematical vocabulary.<br><br>Justify the solution to a system of linear equations algebraically.<br><br>Determine the most efficient way to solve problems involving systems of equations.<br><br>Justify the reasonableness of the solution to a system of equation in context. |

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| | given a pair of coordinates. | system they represent when given two pairs of coordinates. | |

## Instructional Focus Statements

**Level 3:**

In grade 7, students used linear equations in one-variable to solve real-world and mathematical problems (standard 7.EE.B.4). Students also defined the meaning of the variable and explained the solution in terms of the given context. In grade 8, students should build on that knowledge to solve linear equations in one variable and determine if the equation will have one solution, infinitely many solutions, or no solutions (standard 8.EE.C.7). This prior knowledge can be applied to help achieve mastery of this standard which requires students to explore systems of equations, where they solve two equations simultaneously, understanding that a system of linear equations can also have one, many, or no solutions.

Students should be challenged to solve a system of linear equations graphically, algebraically, or by inspection. It is critical that students are able to work flexibly between the different representations and make the explicit connections between the system of linear equations graphically and algebraically. The initial use of graphs will help students to see that the solution to a system is not only the point of intersection, but that it also satisfies both equations when substituted for the variables. Discussions should include careful inspection of the graph to allow students to see that the solution, or point of intersection, is an ordered pair that lies on both lines. Students should be exposed to system of linear equations with lines that intersect, but also with lines that are parallel. Using these visuals, discussion should focus on understanding that parallel lines will never intersect, resulting in no solution. Students should be exposed to equations that are written in both standard and slope-intercept form. Instruction should include pairs of equations that are equivalent, so that students can use previous knowledge about rewriting equations to reveal that they are in fact the same line and discuss why they have infinitely many solutions.

Students know that the slope of a line describes its rate of change (standard 8.EE.B.5) and the slope is the same between any two distinct points on a non-vertical line (standard 8.EE.B.6). Applying this knowledge, students should be pressed to describe the lines represented by pairs of coordinates and/or write the equations of the lines that include these points. Inspection of coordinates and equations will support students' ability to reason about the number of solutions as they should determine that lines with different slopes will have one solution, lines with the same slope and different y-intercepts have no solutions, and lines that have the same slope and same y-intercept will have infinitely many solutions.

Students should have opportunities to work with equations and context that include rational numbers in a real-world situation. Students should be expected to define variables and write pairs of linear equations in two variables to represent a given context. While students may be more comfortable solving systems of linear equations graphically, they should be presented with systems whose solutions do not have integer coordinates. Knowing they can only estimate the solution, students should understand the importance of determining an exact solution by using algebraic methods of elimination or substitution. As with any contextual situation, students should be able to explain and connect the solution back to the context. A strong understanding of systems of linear equations and their solutions will support students as they continue this learning in high school and go on to work with systems of linear inequalities (HS standard A1.A.REI.D.7 and standard M1.A.REI.C.5).

**Level 4:**

At this level, students with a strong, conceptual understanding of systems of linear equations should be able to make connections about the number of solutions a system would have by examining the graph, equations, or pairs of coordinates and explain these relationships using precise mathematical language. Anytime students at this level identify a system of equations has no solution, they should be able to explain, using precise mathematical language, the reason for this and justify their answer graphically and algebraically. When identifying that a system has one solution and solving for that solution, Students at this level should also be expected to justify the solutions using substitution to prove that the solution makes both equations true. While students should know how to solve systems of equations graphically and algebraically, they should be challenged to inspect the system and determine which method (graphical, inspection, or algebraically) would be more efficient and justify their reasoning.

Students with a deep level of understanding can solve systems of equations created from a context and justify their solutions by connecting the solution back to the text. Discussion could focus on having students not only justify the solution mathematically, but recognize the reasonableness of the solution in terms of the context.

# Functions (F)

**Standard 8.F.A.1 (Major Work of the Grade)**

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in 8th grade.)

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Determine that a relation is a function or not a function given a set of ordered pairs or a table. | Explain that a function is a rule that assigns to each input exactly one output and justify their thinking using either a set of ordered pairs, a table of values, or a graph.<br><br>Determine that a relation is a function or not a function given a graph. | Explain that a function is a rule that assigns to each input exactly one output and justify their thinking using a set of ordered pairs, a table of values, and a graph.<br><br>Determine that a relation is a function or not a function. | Explain why a relation sometimes is and sometimes is not a function.<br><br>Create a function or non-functional relationship and provide justification.<br><br>Make connections between functions and non-functional relationships. |

## Instructional Focus Statements

**Level 3:**

In this standard, students are introduced to functions as rules that assign exactly one output to each input. This is the first formal introduction of functions to students. Informally, in previous grade levels, students develop foundational understandings by generating patterns that follow a given rule. In grades 6 and 7, students work with ratios and proportional relationships using tables, equations, and graphs. In grade 8, students should develop a firm foundational understanding that functions describe situations in which one quantity is determined by another. When describing the relationship between input and output quantities, students develop an understanding that the defining characteristic of a function is that the input value determines the output value, or vice versa, that the output depends upon the input value. Students should be able to reason whether a relation presented as a table, graph, or set of ordered pairs models a function or not. To contextualize the concept of a function, it is often beneficial to relate a function to a tool that allows a number to be put in and only allows one number to be put out. It is important that students be allowed to develop a conceptual understanding of the concept of which relations can be defined as functions as opposed to being presented with a set of rules applicable to a particular relation representation.

For example, the vertical line test should result from discovery learning as opposed to being presented as a rule to determine if a graph is a function or not.

**Level 4:**

As students extend their foundational understanding of functions, they should begin to make generalizations about ordered pairs represented in tables, graphs, and equations.  For example, students should discover through repeated reasoning (MP 8), that a relation represented in a graph where two or more x coordinates are the same for any of the points is never a function since this would contradict the definition of function.  These generalizations should be made with verbal and written form justifications and supported with visual representations.

**Standard 8.F.A.2 (Major Work of the Grade)**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and another linear function represented by an algebraic expression, determine which function has the greater rate of change.*

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Determine which function has the greater rate of change, given two linear functions both represented graphically. | Compare properties of two functions, each represented in the same way algebraically, graphically or numerically in tables.<br><br>Determine which function has the greater rate of change, given two linear functions both represented algebraically in slope-intercept form. | Compare properties of two functions, each represented in different ways algebraically, graphically, numerically in tables, or by verbal descriptions. | Compare properties of two functions, each represented in different ways, when the functions are embedded in a contextual problem.<br><br>Compare properties of multiple functions, each represented in different ways algebraically, graphically, numerically in tables, or by verbal descriptions.<br><br>Create contextual situations that involve comparing the properties of two functions each represented in different ways and use precise mathematical language to describe the comparisons. |

**<u>Instructional Focus Statements</u>**

**<u>Level 3:</u>**

Previously, in grade 7, students recognized an important type of regularity in numerical tables by discovering the multiplicative relationship existing between each pair of values in tables generated from equations in the form y = cx and further identifying the constant of proportionality. As students enhance their understanding of functions in grade 8, they should be able to compare properties represented in the same and different forms using tables, graphs, equations, and contextual situations.  To further develop the idea of rate of change (slope), students should be able to compare the rate of change of two functions in multiple representations.  These comparisons should be explained and support with verbal and written explanation as well as visual representation is different forms.  It is also important to note that there are other properties that can be compared beyond the rate of change.  Though an important relationship that exists within functions, rate of change should not be the exclusive focus for this standard.

**Level 4:**

Students should solidify their understanding of functions and comparisons of properties in functions in contextual problems.  Students should be able to deconstruct a contextual situation that involves comparing the properties of two functions and represented each in different ways, providing a justification on why they have represented the function in a certain way to elicit certain properties.  This should be done using precise mathematical language to describe the comparisons.

**Standard 8.F.A.3 (Major Work of the Grade)**

Know and interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

### Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Determine if a function is linear when a graph is provided.<br><br>Determine if a function is non-linear when a graph is provided. | Determine if a function is linear when an equation is provided.<br><br>Determine if a function is non-linear when an equation is provided. | Distinguish between a linear function in the form $y = mx + b$ and a non-linear function.<br><br>Provide examples of linear and non-linear functions. | Explain why a function is linear or non-linear.<br><br>Explain the similarities and differences between linear and non-linear functions in both verbal and written form providing examples of both to justify their thinking.<br><br>Re-write the equation in slope – intercept form and identify that it is a linear function and subsequently graph as a straight line when given a linear equation not in slope-intercept form. |

### Instructional Focus Statements

**Level 3:**

The focus of this standard is for students to become familiar with the slope-intercept form of a linear equation ($y = mx + b$) as defining a linear function that will graph as a straight line. Through repeated reasoning, students should discover similarities and differences of linear and non-linear functions. Linear functions are a major focus of this standard, but students are also expected to give examples of functions that are not linear.

**Level 4:**

As students discover similarities and differences of linear and non-linear functions, they should make generalizations about each and explain their thinking using precise mathematical language accompanied by visual representations to support their reasoning. Additionally, as the standard indicates students should know the equation $y = mx + b$ as defining a linear function, students should be able to make the connection that x and y are the input and output values, $m$ is the multiplicative relationship, and $b$ is the constant or initial value. It is also important for future course work with parent functions that students understand that the "b" (initial value) is a constant value that indicates the number of units the linear function shifts vertically.

**Standard 8.F.B.4 (Major Work of the Grade)**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ($x$, $y$) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Determine the rate of change and initial value when given the equation of a linear function in the form $y = mx + b$.<br><br>Choose a function that models a linear relationship when provided a graph. | Interpret the rate of change and initial value in terms of a situation when given the equation of a linear function in the form $y = mx + b$.<br><br>Determine the rate of change of a function from a graph values.<br><br>Determine the rate of change of a function from two (x. y) values.<br><br>Determine the initial value of a function from a table or graph that contains the initial value. | Construct a function to model a linear relationship between two quantities.<br><br>Determine the rate of change and initial value of a linear function when given a table.<br><br>Determine the rate of change and initial value of a linear function when given a graph.<br><br>Determine the rate of change and initial value of a linear function when given two (x, y) values.<br><br>Interpret the rate of change and initial value of a function in terms of the situation it models. | Create a function to model a linear relationship representing a contextual situation and interpret the rate of change and initial value in terms of the situation it models.<br><br>Make connections about the rate of change when a function is represented in a table, graph, algebraic form, or by verbal descriptions.<br><br>Create a real-world problem which can be modeled by a linear relationship whose solution requires either an interpretation of the rate of change or initial value in terms of the situation it models. |

## Instructional Focus Statements

**Level 3:**

The instruction of this standard should be focused around increasing a student's conceptual understanding of linear functions by building on the knowledge students have from working with the other function standards in grade 8. The end goal is for students to extend their understanding of linear functions to work with linear functions embedded in contextual situations.  Students should be able to construct linear functions from information presented in a wide variety of ways.  Crucial to this is a student's ability to calculate the slope of a line from information presented in a wide variety of ways and to identify the initial value regardless of how information is presented.  Both are imperative as students progress to writing the linear function represented.  Students should also discover the connections that exist between the graphed y-intercept and the initial value of the function. Ultimately, students should be able to write linear functions whose information is embedded in a contextual situation and then explain the meaning of slope and the y-intercept in contextual problems using precise mathematical vocabulary.

**Level 4:**

As students deepen their understanding of constructing functions, they should also be able to create a function to model a linear relationship representing a contextual situation and interpret the rate of change and initial value in terms of the situation it models.  As students extend their learning, they should be able to make connections about the rate of change and y-intercept when a function is represented in different forms, including in a table, graph, equation, or by verbal description.

**Standard 8.F.B.5 (Major Work of the Grade)**

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Determine if a functional relationship is linear or non-linear, given a graph, | Determine if a linear function is increasing or decreasing, given a graph. | Qualitatively describe the functional relationship existing between two quantities when given a linear or non-linear graph.<br><br>Sketch a graph that represents a function that has been described verbally and label the axes appropriately. | Create contextual situations that models a relationship between two quantities and describe qualitatively the functional relationship between the two quantities. Sketch the graph, labeling the axes appropriately.<br><br>Determine the functional relationships that exist for each piece, given a piecewise graph (i.e. which portions of the graph represent a linear relationship, which represent a non-linear relationship, which are increasing, and/or which are decreasing). |

## Instructional Focus Statements

**Level 3:**

The instructional focus of this standard should be helping students understand the connection that exists between a verbal description of important traits of a function to the graph of the function. This should include where the function is increasing or decreasing, and linear or nonlinear. Students should also be able to label and explain the axes with respect to the provided verbal description. Similarly, students should be able to sketch a graph that exhibits the qualitative features of a function given a verbal description. As students develop a conceptual understanding of relationships between quantities and describing the relationships qualitatively, students should pay close attention to the shape of the graph rather than the specific numerical values and analyze the qualitative attributes from left to right describing what happens to the output as the input increases.  A common example is a verbal description or graph of a plane that travels from one point to another where the x-axis indicates the height of the plane and the y-axis indicates time. Students should use precise mathematical language when describing these relationships.

**Level 4:**

To enhance understanding, students should be able to create their own contextual situations that model a relationship between two quantities.  They should describe qualitatively the functional relationship between the two quantities, attending to precision (MP 4), using mathematical vocabulary.  As students sketch graphs to model the contextual situation, they should accurately be able to label the axes and explain what each means with respect to the context.  To strengthen understanding, students should be able to assign specific numerical values to the graph, representing a contextual situation, and provide an explanation of the range indicating where the graph is specifically increasing and decreasing.

## Geometry (G)

**Standard 8.G.A.1 (Supporting Content)**
Verify experimentally the properties of rotations, reflections, and translations:
**8.G.A.1a** Lines are taken to lines, and line segments to line segments of the same length.
**8.G.A.1b** Angles are taken to angles of the same measure.
**8.G.A.1c** Parallel lines are taken to parallel lines.

### Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Define angle, line segment, line, and parallel lines.<br><br>Accurately measure angles and line segments.<br><br>Define rotation, reflection, and translation. | Identify a single transformation when shown on a coordinate plane.<br><br>Explain why a rotation, reflection, or translation of a line segment or angle does not affect the measurements of the line segment or angle.<br><br>Translate lines and line segments on the coordinate plane. | Transform figures on the coordinate plane using rotations, reflections, and translations.<br><br>Use the correct notation when labeling or describing a transformed figure.<br><br>Verify the transformations used when transforming one figure onto another using manipulatives or on the coordinate plane.<br><br>Verify that angle measures and lengths of line segments remain the same after translations, rotations and reflections.<br><br>Verify that parallel lines remain parallel after translations, rotations and reflections. | Develop multiple solution paths that represent a series of transformations from one figure onto another.<br><br>Use precise mathematical language when explaining a complex series of transformations. |

## Instructional Focus Statements

**Level 3:**

In grade 7, students learned about dilations to reproduce scale drawings in mathematical and real-world problems (standard 7.G.A.1). Students should add rotations, reflections, and translations to their study of transformations. Students should spend time exploring these transformations and verify through experimentation and discovery with figures on a coordinate plane including straight lines and line segments, as well as congruent angles and parallel lines. Students should have ample opportunities to explore and learn about rotations, reflections, and translations on both a coordinate plane as well as with real-world examples. Students should have ample opportunities to experiment with translations, rotations and reflections using transparencies, tracing paper and/or graph paper, compasses, protractors, rulers, and technology. Discussion should focus on what students notice about the new figure in comparison to the original. Students should engage in discourse around the characteristics of figures (i.e., angle measures, parallel lines, lengths of line segments) before the transformation (pre-image) and after the transformation (image). This should lead to an understanding that these transformations produce images of exactly the same size and shape as the pre-image, creating congruent figures. Use of correct mathematical vocabulary and notation such as *A* and *A' (A' read as "A prime")* is expected.

Instruction should include opportunities to communicate, both orally and in writing, explanations of the transformation sequence. These discussions should lead students to be able to transform figures on a coordinate plane when given multiple transformation properties as well as identify a series of transformations used when shown a pre-image and an image on a coordinate plane.

**Level 4:**

Students at this level of understanding should be expected to use precise mathematical language to explain a complex series of transformations between an image and its pre-image. Students should also be challenged to develop multiple solution paths to represent a series of transformations from one figure onto another. By giving students two images on the same plane, they could being asked to identify a series of transformations that maps one onto the other. In addition, students would need to define which image was the pre-image to accurately identify the series of transformations. Students could also be challenged to come up with a path both to and from the one image to another, to help students recognize two methods of creating the same set of images.

**Standard 8.G.A.2 (Supporting Content)**

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Differentiate between similar and congruent figures.<br><br>Recognize if an image and its pre-image represent rigid or non-rigid transformations<br><br>Generate figures using coordinates on the coordinate plane. | Identify if an image and a pre-image represent a translation, rotation, reflection, or dilation.<br><br>Use coordinate notation to label the vertices of the transformed image when given a labeled pre-image.<br><br>Identify which transformations maintain congruence and which transformations maintain similarity.<br><br>Describe the translation of an image as the number of units shifted and the direction of the shift. | Describe how reflections affect the coordinates of any image.<br><br>Describe how the rotation affects the coordinates of an image when given a degree of rotation.<br><br>Use coordinate notation to describe the transformation when given an image and its pre-image.<br><br>Identify images that undergo translations, reflections, and/or rotations as congruent figures.<br><br>Identify images that are dilated as similar figures.<br><br>Use coordinates of figures dilated from the origin to identify the scale factor between the image and the pre-image.<br><br>Describe the effect a dilation will have on an image and its coordinates when given the scale factor. | Explain the effects of rigid and non-rigid transformations using coordinates and precise mathematical language.<br><br>Describe the relationship of the figures using transformations when given a set of figures in a context. |

## Instructional Focus Statements

**Level 3:**

The study of the properties of transformations of figures is the focus of standard 8.G.A.1 and is extended with this standard to transformations on a coordinate plane, with emphasis on the coordinates of the resulting image after the transformation(s). By also extending student learning from grade 7, where they were taught to solve and reproduce problems involving scale drawings of geometric figures (standard 7.G.A.1), students should discover dilations as a transformation that creates figures that are not congruent shapes, but rather similar shapes with proportional side lengths.

Students should be given the opportunity to examine several original figures with their figures after transformation. Instruction should include activities where students have time to explore transformations using tracing paper or technology to discover the relationship between the coordinates of the image and the pre-image to develop a sense of how the coordinates change during different transformations. Discussion should help students recognize what patterns emerge in the operations or mathematical changes to the numbers for each type of transformation. Students should describe the transformations leading from a pre-image to a resulting image, recognize the relationship between the coordinates of the image and pre-image, and use proper coordinate notation when labeling them. By comparing the coordinates of given figures, students can be led to discover the relationship of the coordinates, which helps students identify the scale factor in figures that have been dilated from the origin. A common misconception is that students do not multiply all coordinates when dilating, so it is important to give students examples and non-examples to highlight these errors.

In high school, students will advance their understanding of congruence and similarity to develop proofs to determine if two figures are congruent or similar (G.CO).

**Level 4:**

Students at this level should be challenged to explain the effects of rigid (translations, rotations, reflections) and non-rigid transformations (dilations) while using precise mathematical language. When given a pre-image and an image, students should be expected to identify a series of transformations and based on those transformations, be able to justify whether the two images are congruent or similar. Classroom discussion should take place to have students share their thoughts to encourage students to identify and correct mistakes, requiring them to analyze the solution paths of others and justify their reasoning. In addition to examining examples on a coordinate plane, students should be challenged to generate a context for an image and its pre-image using examples found in a real-world setting such as art, architecture and the natural world.

**Standard 8.G.A.3 (Supporting Content)**

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

*For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Recognize vertical, adjacent and supplementary angles.<br><br>Solve for a missing interior angle of a triangle by applying the triangle sum theorem.<br><br>Identify the exterior angles of a triangle.<br><br>Find the measure of a missing angle in a straight angle.<br><br>Identify parallel lines and the intersection of a transversal. | Identify similar triangles based on their relationship if the two triangles have two pairs of congruent angles.<br><br>Identify pairs of congruent angles created by parallel lines and a transversal.<br><br>Determine angle congruence based on the relationship of the angles.<br><br>Informally explain the relationship between interior and exterior angles of a triangle.<br><br>Use facts about angle relationships to find missing interior and exterior angle measures of a triangle. | Informally explain the triangle sum theory using three copies of a triangle.<br><br>Give informal arguments to establish facts about the angle sum of triangles.<br><br>Give informal arguments to establish facts about exterior angles of triangles.<br><br>Informally explain the relationship of angles created by parallel lines cut by a transversal.<br><br>Apply transformations to informally generate arguments for similarity of triangles.<br><br>Justify missing interior and exterior angle measures of a triangle using facts about angle relationships. | Make a generalization for the angle-angle criterion to establish mathematical facts about similar triangles.<br><br>Justify mathematically the relationships of lines and angles created by parallel lines cut by a transversal.<br><br>Solve contextual and mathematical problems involving triangles.<br><br>Solve contextual and mathematical problems involving a pair of parallel lines cut by a transversal.<br><br>Justify missing interior and exterior angle measures of a triangle in an organized table. |

## Instructional Focus Statements

**Level 3:**

In grade 8, students build on their experimentation with angles (standard 7.G.B.4) and triangles (standard 7.G.A.2) to construct informal arguments about their properties while exploring facts about the angles of a triangle, angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similar triangles. Students should have opportunities to engage in meaningful discussions around the relationships between interior and exterior angles as well as use measurement to verify angle pair relationships. Students should be led to consider constraints on angles in triangles through discussion and exploration. Given the opportunity to build triangles, and examine their interior angles of a triangle, students realize the sum of two interior angle measures must be less than 180 degrees to form a third vertex. Opportunities to explore will also promote students' understanding of the triangle sum theorem, leading them to make conjectures about the sum of the three interior angles of a triangle. Using three copies of a triangle, students can arrange their interior angles (at the vertices) to show that they form a straight line with a measure of 180 degrees. Students should also have multiple opportunities to apply what they know about transformations and angle relationships to find missing angle measures when parallel lines that are cut by transversals form a triangle. For example, discussing why corresponding angles are congruent using translations and why alternate interior angles are congruent using rotations. Because students have experimented with transformations, instruction should include activities that require students to apply these understandings to informally proving the angle-angle criterion for similarity of triangles. Rather than just memorizing a rule, students should be encouraged to think about and articulate why the rule can be generalized. Computer software could be used to enhance these opportunities so that students visualize the impact of different transformations. Encouraging students to think strategically and make conjectures about those relationships will help students create informal arguments to establish facts, which will be beneficial when they are expected to use proofs in high school (standards G.CO.9 and G.CO.10).

**Level 4:**

Students at this level of understanding should have instruction that is focused on communicating to explain their mathematical reasoning as facts that have been proven. Students should be able to refute conjectures about angles in triangles and angles formed when two parallel lines are cut by a transversal(s). Students should provide counterexamples with their explanation when disproving these conjectures. These critiques will help students develop a deeper understanding of angles formed with triangles and the need for clear and precise language that they will later apply when they begin to develop proofs. Although two-column proofs are not expected at this grade level, a student at this level could be challenged to start thinking and justifying in an organized way leading to the construction of proofs.

**Standard 8.G.B.4 (Major Work of the Grade)**

Explain a proof of the Pythagorean Theorem and its converse.

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Identify the hypotenuse and legs of a right triangle.<br><br>Define right triangle.<br><br>Represent a number squared using a geometric representation of a square. | Choose a statement to complete a partially written proof.<br><br>Determine if a triangle is a right triangle using the length of the sides.<br><br>Identify the legs and hypotenuse represented in the formula $a^2 + b^2 = c^2$.<br><br>Construct a right triangle given the lengths of the sides. | Use a model to explain the Pythagorean Theorem.<br><br>Justify a triangle as a right triangle using the converse of the Pythagorean Theorem. | Critique the work of others in justifying the Pythagorean Theorem and explain why that work is correct or incorrect.<br><br>Critique the work of others in justifying the converse of the Pythagorean Theorem and explain why that work is correct or incorrect.<br><br>Create examples of right triangles and explain why they are or are not right triangles using the Pythagorean Theorem or its converse. |

## Instructional Focus Statements

### Level 3:

In grade 7, students explored geometric conditions to determine when a triangle could be produced. In grade 8, students extend their learning to explore the relationship between side lengths of right triangles. Instruction should focus on having students explain the Pythagorean Theorem and use its converse to determine if a triangle is a right triangle. It is essential that students be provided opportunities to create and explain models that illustrate the Pythagorean Theorem to discover and show that the area of the square that forms the hypotenuse is equivalent to the sum of the squares that form the legs. Students should be expected to use their understanding of the proof to give meaning to the formula $a^2 + b^2 = c^2$. Likewise, if the square of the longest side (c) of a triangle is equal to the sum of the squares for the other two sides (a and b), then the angle opposite of side c is a right angle and the triangle is a right triangle (the converse statement). Students should have multiple opportunities to engage in discourse around the converse statement and explore with situations that involve triangles that are and are not right triangles. Important vocabulary terms that should be used throughout instruction include: hypotenuse, legs, right angle, square of a number, area of a square, and Pythagorean Theorem. Students should be encouraged to use these terms when describing right triangles. A common mistake for some students is trying to use the Pythagorean Theorem to find missing sides for triangles that are not right triangles, so multiple experiences in testing the theorem with non-right triangles will support students' understanding of how the theorem works. A thorough understanding of the Pythagorean Theorem proof will be beneficial as students prove theorems about similar triangles in high school (standard G.SRT.B.4).

**Level 4:**

Students at this level should be challenged to go from informal explanations to mathematical justifications. Students should not only be challenged to make connections between representations and proofs of the theorem, but also provide feedback to a peer who explains the theorem and its converse. Discussion should be facilitated so that students have opportunities to explain why a triangle is or is not a right triangle. Additionally, their peers should be encouraged to critique their explanations in using precise mathematical vocabulary.

**Standard 8.G.B.5 (Major Work of the Grade)**
Know and apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Identify which side of a given right triangle is the hypotenuse and which two sides are the legs. | Use the Pythagorean Theorem to determine the length of the hypotenuse of a right triangle when a visual representation is provided. | Apply the Pythagorean Theorem to solve real-world or mathematical problems in two dimensions.<br><br>Apply the Pythagorean Theorem to solve real-world or mathematical problems in three-dimensions when a visual representation is provided. | Apply the Pythagorean Theorem to solve real-world or mathematical problems in three dimensions.<br><br>Apply the Pythagorean. Theorem to real-world and mathematical problems in two dimensions from increasingly complex situations. |

## Instructional Focus Statements

**Level 3:**

Students should be exposed to visual representations that lead them to the conclusion that for right triangle $a^2 + b^2 = c^2$, where a and b are leg lengths and c is the hypotenuse. For problems eliciting the use of the Pythagorean Theorem, students should be exposed to a balance of problems between finding unknown leg lengths and finding an unknown hypotenuse length.

Students may also need to review solving equations where they must use square roots. It is not necessary at this grade for students to simplify square roots.

Students should draw diagrams from contextual situations in order to visualize the application of the Pythagorean Theorem. Three-dimensional representations are very difficult for students to visualize. Students should practice drawing and labeling the right triangle that exists in three-dimensional objects in order for them to solidify this understanding. Using three-dimensional physical models may help with developing student understanding.

**Level 4:**

Students should demonstrate an understanding of the Pythagorean Theorem by describing mathematical and real-world situations in which it can be applied. As students are applying the Pythagorean Theorem to complex problems, they should provide valid justification of their reasoning using precise vocabulary. Additionally, students should be able to visualize three dimensional objects with embedded right triangles without visual representations provided.

Standard 8.G.B.6 (Major Work of the Grade)

**Standard 8.G.B.6 (Major Work of the Grade)**

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

## Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
| --- | --- | --- | --- |
| Create a right triangle from two given points on the coordinate plane.<br><br>Evaluate the square root of perfect squares. | Identify the legs and the hypotenuse for any right triangle on a coordinate plane.<br><br>Approximate the value of non-perfect square roots using a calculator or by estimating what two whole numbers it would lie between.<br><br>Determine the measure of each leg for any right triangle on a coordinate plane. | Find the distance between two points on a coordinate plane using the Pythagorean Theorem.<br><br>Apply the Pythagorean Theorem to right triangles on a coordinate plane. | Use the Pythagorean Theorem and the coordinate plane to solve contextual problems.<br><br>Explain solution strategies using precise mathematical vocabulary.<br><br>Make explicit connections to the Pythagorean Theorem when given the distance formula |

## Instructional Focus Statements

**Level 3:**

Instruction should build on work from grade 6 where they found distances between vertical and horizontal lines on the coordinate plane (standard 6.G.A.3). This idea should be extended to finding the distance between two non-vertical and non-horizontal points by applying the Pythagorean Theorem. When given two points, instruction should lead students to recognize a diagonal line to connect the points, and discussion should lead students to see how they can form a right triangle by using the coordinate plane. Students can then build on their knowledge and understanding of the Pythagorean Theorem from standard 8.G.B.5 and apply these concepts to the triangles created on the coordinate plane. It is imperative that students recognize the diagonal as the length of the hypotenuse and the horizontal and vertical lines on the coordinate plane as the legs of the right triangle. To foster a better understanding of distance, students may benefit from initially measuring the triangle's side lengths. Students should be exposed to problems where they approximate the length of an unknown side of a right triangle even when the length is not a whole number.

**Level 4:**

At this level of understanding, students should be pressed to extend their mathematical learning to situations where they would utilize the coordinate plane as a tool and the Pythagorean Theorem as a strategy to solve contextual problems. Students should focus on using precise mathematical language to explain their solution strategies to others. Although the distance formula is not expected at this level, a way to challenge students is to provide them with the distance formula and let them discover and explain how the formula is derived from the Pythagorean Theorem. Practice with triangles on the coordinate plane will prepare students to apply the distance formula in high school.

**Standard 8.G.C.7 (Supporting Content)**
Know and understand the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Find the volume of a cone, cylinder, or sphere, given the formula and visual model. | Recognize the formula needed to find the volume of a cone, cylinder, or sphere and use it to find the volume when a visual model is provided. | Apply volume formulas to solve real-world or mathematical problems involving cones, cylinders, and spheres. | Apply volume formulas to solve complex real-world or mathematical problems. |

## Instructional Focus Statements

**Level 3:**

Students begin to understand two- and three-dimensional shapes in elementary grades and should build on these to develop a conceptual understanding of the volume of cylinders, cones, and spheres. Students should extend their knowledge of the area of a circle to make a connection that the volume of a cylinder is the area of a circle, or the cylinder's base, multiplied by the height resulting in the formula $V = \pi r^2 h$ . Students should also connect the volume of a cylinder to prior knowledge of the volume of a right rectangular prism, as both volumes are the area of the base multiplied by the height.

When developing the conceptual understanding for the volume of a cone, students should use visuals or models to make the connection that the volume of a cone is a third of the volume of a cylinder with the same height, thus resulting in the volume of a cone formula of $V = 1/3\ \pi r^2 h$. As students begin to understand the volume of a sphere, they should make the connection that a sphere enclosed in a cylinder, where the diameter is equivalent to the height of the cylinder, is 2/3 the volume of the cylinder. Then, by substituting the diameter of the sphere, 2r, for the h in the formula for the volume of the cylinder, students will discover the resulting volume of a sphere formula of $V = 4/3\ \pi r^3$.

Additionally, it is imperative for students to make connections to previous geometric coursework to develop an in-depth conceptual understanding of the volume formulas to apply to real-world and mathematical problems.

**Level 4:**

Students should demonstrate their conceptual understanding of the volume of cylinders, cones, and spheres by making connections to previous geometric coursework, explaining their reasoning in both verbal and written form using precise mathematical vocabulary. Students should also be able to clearly recognize which formula is needed to find the volume in solving complex real-world or mathematical problems. Additionally, students should generalize their conceptual understanding and connections of the volume formulas and use them to efficiently solve real-world and mathematical problems.

## Statistics and Probability (SP)

**Standard 8.SP.A.1 (Supporting Content)**
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

### Evidence of Learning Statements

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Identify any clusters and outliers represented on the graph when given a scatterplot.<br><br>Identify positive or negative association represented on the graph when given a scatterplot.<br><br>Explain the meaning of the values of a given ordered pair in a context. | Determine if a scatter plot has linear, nonlinear, or no association.<br><br>Identify clusters, outliers, and gaps in data in a given scatter plot.<br><br>Define "bivariate measurement". | Construct scatter plots using two-variable data sets<br><br>Describe patterns of association for two-variable data sets represented in scatter plots.<br><br>Identify the relationship of the two quantities being represented by a scatter plot in context. | Create a context to describe bivariate data from a given scatter plot.<br><br>Interpret a scatter plot and make predictions based on the patterns of association using precise mathematical language.<br><br>Describe what clusters and outliers reveal about the data from a scatter plot. |

### Instructional Focus Statements

**Level 3:**

In grades 6 and 7, students modeled relationships between quantities the coordinate plane. Instruction should build on that understanding as students are asked to model and interpret scatterplots and describe the associations represented in their scatterplots or given scatterplots. Students should have opportunities to extend their understanding of linear relationships to investigate patterns of association between the quantities. Students should be expected to recognize and explain the meaning of positive and negative correlations, clusters, and gaps in data. Discourse should include opportunities to describe outliers as deviations from associated data, as well as opportunities to describe linear, and non-linear trends. Additionally, instruction should include problems where no trends are evident. Discussions should include examples that lead students to understand that a trend in a scatter plot does not indicate cause and effect.

Through the course of learning students begin to realize that constructing a scatter plot can make it easier to identify when relationships between bivariate measurements occur. Students should also have opportunities to explore and practice constructing graphs by hand, using calculators, or through the use of computer software programs. In high school students will continue their learning of bivariate data using two-way frequency tables where they will interpret trends within the context of the data.

**Level 4:**

To deepen understanding, students at this level should be given opportunities to create contextual problems that could be used to represent a given set of data.

Opportunities for students to gather their own bivariate data for representation and analyzation should be offered for students at this level. Students could be challenged to conduct research to collect bivariate data or gather their own data and then generate a scatter plot, make predictions about the data, and summarize their findings using precise mathematical vocabulary.

**Standard 8.SP.A.2 (Supporting Content)**
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Identify features of a linear relationship.<br><br>Determine if a scatter plot has linear association. | Determine if a scatter plot has positive, negative, or no relationship between two quantities.<br><br>Draw a straight line that closely fits the data points on a scatter plot. | Construct a table of values, plot points, and connect points to model linear relationships in context.<br><br>Determine which line best models the association of the data when given a scatter plot with various possible lines of fit.<br><br>Determine the accuracy of a line of fit based on the closeness of the data points to the line. | Determine the mean of the x-values and y-values on a scatter plot to identify the centroid point on the estimated model line of fit.<br><br>Explain the meaning of the line of fit and its properties using precise mathematical language in terms of the context.<br><br>Critique the accuracy of a line of fit to a data set represented in a scatterplot. |

## Instructional Focus Statements

**Level 3:**

In standard 8.SP.A.1, students are expected to describe qualitative bivariate data that is represented on a scatter plot. From that standard, essential vocabulary terms such as outliers, clusters, positive, negative, strong, weak, linear correlation, and non-linear correlation are used to describe key features of scatter plots. As a progression, students now determine when a linear relationship is present in a scatter plot and explain the relationship if one exists. Students should be exposed to strong and weak examples of linear and non-linear relationships and engage in discourse around the fit of the line.

Discussion of a variety of graphs should lead students to the understanding that a straight line can model a relationship of the data points when there appears to be a linear association. When a linear relationship exists, students should realize that the closer the points are to a line, the stronger the linear relationship. Students might struggle to draw a straight line, so a ruler or other straight edge could be used to help students decide where to place their lines. Transparent rulers could be beneficial for students because they can see the points while finding the best place for their line. When students initially begin fitting lines to data, instruction could begin by providing them with pre-plotted scatter plots so that the focus is on drawing the line based on the trend represented by the data points.

Students should have opportunities to compare lines of best fit when given the same set of data. A common misconception for students is thinking that their lines of best fit for the same set of data will be or must be exactly the same, not understanding that the lines of best fit are informally drawn to approximate an equation and represent an association between two data sets. Instruction should facilitate open discussion with students and give opportunity for students to present multiple lines of fit and compare all options. This gives the opportunity to clarify any informal understandings that students may apply in the construction of the line and solidify the understanding that the most appropriate line is the one that comes closest to most data points, and therefore the line may or may not go through any or all of the data points.

**Level 4:**

Students with a solid understanding of lines of fit should be expected to explain, both orally and in writing, the meaning of the line of best fit. Instructional should challenge students to begin to make more precise decisions and justify their answers using precise mathematical language. Students should also be given the opportunity compare and critique suggested lines of fit. One method of justification students at this level could be introduced to would be determining the centroid point for the line by calculating the mean of the x-values and y-values. In addition, students should be expected provide a detailed explanation of this process as well as the effect of any outliers may have, using precise mathematical vocabulary.

**Standard 8.SP.A.3 (Supporting Content)**

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Determine the slope and the y-intercept for a line that is graphed on a coordinate plane.<br><br>Describe the relationship between variables on a graph in context.<br><br>Write the equation of a line given the slope and y-intercept. | Draw a line of best fit for bivariate measurement data.<br><br>Write an equation for the line of best fit on a scatter plot. | Use a linear model to solve contextual problems.<br><br>Interpret the slope of a linear model in context of bivariate measurement data.<br><br>Interpret the y-intercept of a linear model in context of bivariate measurement data. | Make predictions using a linear model and describe the accuracy of the predictions in the given context.<br><br>Conduct an experiment to gather, graph, and write a linear equation for bivariate data.<br><br>Assess the reasonableness of using the point on a linear graph in the context of the situation. |

## Instructional Focus Statements

**Level 3:**

Students extend their understanding from grades 6 and 7 where they graphed and created equations of quantitative relationships. Additionally, students interpreted the meaning of points (x, y) and rates within proportional relationships and eventually applied this same understanding to non-proportional relationships.

Instruction should build on students' experience of modeling linear relationships by constructing scatter plots and lead them to solve authentic problems involving bivariate data using linear equations. In addition to writing equations that represent the line fit to the data, discussion should go beyond identifying and should focus on the meaning of the slope and y-intercept of the line in terms of the context. Students should be exposed to a variety of scatter plots, where the points are coplanar, but generally non-collinear, and be able to select appropriate points in order to write the equation of the line.

A variety contextual questions should be presented to allow students to realize that the equation of the line of best fit can be used to solve these problems. Discourse should include opportunities for students to explain how they determined the equation of the line and how closely values found using the equation are to the actual values in the given data set. A solid understanding of linear models will be essential as students move beyond linear models and explore quadratic and exponential models in high school.

**Level 4:**

At this level of understanding, students can be challenged to take on more ownership of the entire process of analyzing bivariate data. Opportunities should be given for students to hypothesize a linear relationship, gathering their own data, creating their own graph and line of best fit, and writing an equation for the linear association they have found in their experiment. Additional elements of these activities could include giving a presentation that describes and interprets the slope and intercept for others as they present their findings or giving students the opportunity to interpret the slope and y-intercept of represented in the linear models of others. Students should be able to debate the reasonableness of some points that may fit the line and justify why these points would not make sense due to the context of the situation. For example, although a point on a line may represent 3.6 this would not make sense if our data is about people, whereas it would make sense if we were talking about a measurement.

To further prepare students for models of other functions, instruction can also lead students to use the best fit line to make predictions for what is likely to happen in a given situation. Students should be encouraged to analyze their own predications and critique the predications of others for reasonableness in terms of the contexts given.

**Standard 8.SP.A.4 (Supporting Content)**

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

**Evidence of Learning Statements**

| Students with a level 1 understanding of this standard will most likely be able to: | Students with a level 2 understanding of this standard will most likely be able to: | Students with a level 3 understanding of this standard will most likely be able to: | Students with a level 4 understanding of this standard will most likely be able to: |
|---|---|---|---|
| Determine the probability of a simple event.<br><br>Understand that a probability is between 0 and 1, and that something unlikely to happen has a probability closer to 0, something likely to happen has a probability closer to 1, and something equally likely to happen is halfway between 0 and 1.<br><br>Express the probability of a single event using the appropriate terms impossible, unlikely, equally likely, likely, or certain. | Express probability of a simple event as a fraction, decimal and/or percent.<br><br>Differentiate between simple events and compound events.<br><br>Differentiate between theoretical and experimental probability.<br><br>Estimate the probability of an event by collecting data and observing its long-run frequency. | Determine the sample space of a compound event.<br><br>Use probabilities to make decisions in real-world situations.<br><br>Recognize that the number of possible outcomes for a compound event is determined by multiplying the number of outcomes for each individual event.<br><br>Determine the probability of compound events using lists, tables, tree diagrams, and simulations.<br><br>Compare compound probabilities that are based on theoretical models with experimental probability simulations.<br><br>Express the probability of a compound event as a fraction, decimal, and/or percent. | Determine whether or not a given probability model is plausible and justify your explanation.<br><br>Explain how a given contextual situation models a compound event and how the probability can be approximated.<br><br>Design a simulation and use the results to estimate a probability. |

## Instructional Focus Statements

**Level 3:**

In grade 7, students explored the probability of simple events by developing models and comparing them with experimental probability (standard 7.SP.C). Students in grade 8, will build on their knowledge of single events to determine the probability of compound events. As they did with simple events, students should have opportunities to conduct experiments using a variety of random generation devices (e.g., spinners, number cubes, coin toss, etc.). Instruction should include opportunities to interpret and create probability models that illustrate possible outcomes and sample space for compound events. Visual models will be helpful as some students might be tempted to add, rather than multiply, when finding the probability of these events. Opportunities for discourse will allow students to compare simple events to compound events and explain, both orally and in writing, similarities and differences between the two using appropriate probability terms. Instruction should lead students to recognize the benefits of using organized lists, tables, and tree diagrams when representing sample spaces for compound events.

**Level 4:**

Students at this level should go beyond calculating probabilities, to using probabilities to make informed decisions about real-world situations. Students should design their own simulations and use the results to estimate and make theoretical predications based on their experiments. Students should also be able to explain using precise mathematical vocabulary why or why not a given probability model is valid based on the context.